Qubit-efficient entanglement spectroscopy using qubit resets

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• Design new NISQ algorithms for entanglement spectroscopy.
  • Task: given copies of $|\psi\rangle_{AB}$ and a parameter $n$, estimate $\text{Tr}(\rho_A^n)$.
  • Require **asymptotically fewer qubits** than any previous efficient algorithm; but still similarly noise-resilient.
  • Key tool: qubit reset
  • Test using numerical simulations and on Honeywell System HØ.

• Define *effective circuit depth* to explain results and analyze future algorithms using qubit resets.

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Entanglement spectroscopy

• At a high level, goal is to understand the entanglement of a state $|\psi\rangle$.
  • e.g. $|\psi\rangle$ is output of some quantum simulation

• In particular, understand the bipartite entanglement of $|\psi\rangle_{AB}$ on systems $A$ and $B$.
  • Fully characterized by the eigenvalues of reduced state $\rho_A = \text{Tr}_B(|\psi\rangle\langle\psi|)$.
  • Learning the spectrum yields more information than entropy alone.
  • Important for understanding topological order, phases transitions, whether a system obeys an area law (and thus can be simulated classically), ...
Entanglement spectroscopy

• Formally: Given as input a parameter $n$ and black-box access to a circuit preparing $|\psi\rangle_{AB}$, estimate $\text{Tr}(\rho_A^n)$.
  • Note not $\text{Tr}(\rho_A \otimes^n)$.

• The first few traces often can be used to reconstruct the largest few eigenvalues of $\rho_A$, which are often sufficient for applications.
  [Li, Haldane 08], [Johri, Steiger, Troyer 17], ...

• Directly related to $n$-th Rényi entropy: $S_n(\rho) = \frac{1}{1-n} \log(\text{Tr}(\rho^n))$. 
Previous algorithms for $\text{Tr}(\rho_A^n)$

- Two known efficient, NISQ-friendly algorithms:
  - HT – a variant of the Hadamard Test
    [Johri, Steiger, Troyer 17]
  - TCT – a variant of the Two-Copy Test
    [Subaşı, Cincio, Coles 19]
Previous algorithm #1 for $\text{Tr}(\rho_A^n)$

**The HT**

- Recall SWAP Test:
  - When $\rho = \sigma$, gives
    \[
    \text{Tr}(\text{SWAP} \rho \otimes \rho) = \text{Tr}(\rho^2)
    \]

- Generalize [Johri, Steiger, Troyer 2017]:
  - Use Cyclic Permutation operator $P_A^{\text{cyc}}$ on $n$ copies of $|\psi\rangle$ to compute $\text{Tr}(\rho_A^n)$.

**HT =**

Cyclic Permutation: e.g. $1234 \rightarrow 4123$
Previous algorithm #2 for $\text{Tr}(\rho^n_A)$

The TCT

- Bell Basis ($|\phi^+\rangle, |\phi^-\rangle, |\psi^+\rangle, |\psi^-\rangle$) is an eigenbasis of SWAP.

$\rightarrow$[Garcia-Escartin, Chamorro-Posada 2013] [Cincio, Subaşı, Sornborger, Coles 2018]
Can measure state overlap using a Bell Basis Measurement.

$\bullet$ [Subaşı, Cincio, Coles 2019]
Measure overlap of $|\psi\rangle^\otimes n$ and $P_A^{cyc} |\psi\rangle^\otimes n$

$$\left| \langle \psi |^\otimes n P_A^{cyc} \right| |\psi\rangle^\otimes n \right|^2 = \text{Tr}(\rho^n_A)^2$$

$\rightarrow$ $\text{Tr}(\rho^n_A)$
Previous algorithm #2 for $\text{Tr}(\rho^n_A)$

The TCT

[Subaşı, Cincio, Coles 2019] Using Bell-basis Measurement to measure overlap of $|\psi\rangle^\otimes n$ and $P_{A}^{\text{cyc}} |\psi\rangle^\otimes n$ to estimate $\text{Tr}(\rho^n_A)$

• **Neat trick**: Apply $P_{A}^{\text{cyc}}$ without any gates. Just reindex the CNOTs and the postprocessing formula.

• $O(1)$ depth: 1 layer of CNOT, 1 layer of H
Our qubit-efficient algorithms

• Where $|\psi\rangle$ is 2$k$ qubits,
  • HT requires $2nk = O(nk)$ qubits
  • TCT requires $4nk = O(nk)$ qubits

• We give variants of HT and TCT to compute $\text{Tr}(\rho_A^n)$ that require $O(k)$ qubits (as few as $3k + 1$).
  Independent of $n$. An asymptotic difference.

• Our new, low-width circuits have larger depth.
  But, we use qubit resets to avoid the usual noisy affects.
Qubit resets

• We usually think of “resetting” qubits to their $|0\rangle$ state at the beginning & end of a computation.

• “Intermediate measurement and reset” is now being rolled out by Honeywell, IonQ, IBM, etc.

• Can reset, i.e. drive back to $|0\rangle$ state, individual qubits in time comparable to a measurement. Then, reuse it.

• They’re an underexplored tool that will be crucial for NISQ:
  • [Rattew, Sun, Minssen, Pistoia 20] [Foss-Feig et al 20] [Liu, Zhang, Wan, Wang 19] and just a few other examples to date.
Our algorithm: qe-TCT

- A new $|0\rangle$ is a qubit reset.
- Each $|ψ_i\rangle$ is identical; indexing just for convenience
- Quick check: each copy only interacts with two other copies.
- **Key intuition**: When a register finishes its interactions, measure, reset, and reuse to load a new copy. Repeat as necessary for $n$.
- So, number of qubits to compute $\text{Tr}(ρ^n_A)$ is independent of $n$
Numerical simulations

• Tested our algorithms and previous algorithms under simulated gate, thermal, and readout noise (see paper for noise parameters).

• Input different $|\psi\rangle$ with varying entanglement. For each value of $n$, use slope of true $\text{Tr}(\rho_A^n)$ vs estimated $\text{Tr}(\rho_A^n)$ to determine quality of algorithm. We plot the slopes.

• **Takeaway**: The new algorithms perform similarly to original.

HT and qe-HT  TCT and qe-TCT
Honeywell system HØ

- Tested on Honeywell 6-qubit ion trap quantum computer.
- We ran qe-TCT on three different states up to $n = 7$. The original TCT would require 28 qubits, more than the 6 available.
Effective depth

- Circuit depth is interesting because:
  1. Able to prove things about bounded-depth circuits like AC₀, QNC₀, etc.
  2. It’s a good heuristic for susceptibility to thermal relaxation and decoherence noise: i.e., deeper circuits perform worse.

- Depth is a bad heuristic for circuits using resets.
  - Evidence: Our circuits!

- Original TCT has $O(1)$-depth
- qe-TCT has $\tilde{\Theta}(n)$-depth
- Asymptotic difference, but they perform similarly.
Effective depth

• Naïve idea: longest time any qubit goes between resets

• Counterexample:
Effective depth

• **Effective circuit depth**: maximum length of a path along which there is information flow.

• Then, both the Original TCT and qe-TCT have *effective depth* equal to $O(1)$.

• Reduces to standard depth for circuits without resets.
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• New algorithms for estimating $\text{Tr}(\rho_A^n)$ which require asymptotically fewer qubits but achieve similar noise resilience. Enable spectroscopy of larger quantum systems on NISQ devices than previously possible.

• **Effective circuit depth** generalizes standard depth to circuits using qubit resets. Useful for predicting noise-resilience of future circuits.

• **Open**: What other algorithms and applications can be made NISQ-ready using qubit resets?