# Qubit-efficient entanglement spectroscopy using qubit resets


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# Qubit-efficient entanglement spectroscopy using qubit resets

- Design new NISQ algorithms for entanglement spectroscopy.
  - Task: given copies of $|\psi\rangle_{AB}$ and a parameter $n$, estimate $\mathrm{Tr}(\rho_A^n)$.
  - Require **asymptotically fewer qubits** than any previous efficient algorithm; but still similarly noise-resilient.
  - Key tool: qubit reset
  - Test using numerical simulations and on Honeywell System HØ.

- Define *effective circuit depth* to explain results and analyze future algorithms using qubit resets.

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# Entanglement spectroscopy

- At a high level, goal is to understand the entanglement of a state $|\psi\rangle$.
  - e.g. $|\psi\rangle$ is output of some quantum simulation
- In particular, understand the bipartite entanglement of $|\psi\rangle_{AB}$ on systems $A$ and $B$.
  - Fully characterized by the eigenvalues of reduced state $\rho_A = \mathrm{Tr}_B(|\psi\rangle\langle\psi|)$.
  - Learning the spectrum yields more information than entropy alone.
  - Important for understanding topological order, phases transitions, whether a system obeys an area law (and thus can be simulated classically), …

# Entanglement spectroscopy

- Formally: Given as input a parameter $n$ and black-box access to a circuit preparing $|\psi\rangle_{AB}$, estimate $\mathrm{Tr}(\rho_A^n)$.
  - Note not $\mathrm{Tr}(\rho_A^{\otimes n})$.

- The first few traces often can be used to reconstruct the largest few eigenvalues of $\rho_A$, which are often sufficient for applications.
  [Li, Haldane 08], [Johri, Steiger, Troyer 17], …

- Directly related to $n$-th Rényi entropy: $S_n(\rho) = \frac{1}{1-n}\log\big(\mathrm{Tr}(\rho^n)\big)$.

# Previous algorithms for $\mathrm{Tr}(\rho_A^n)$

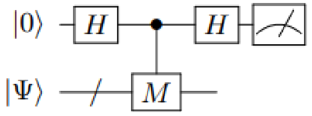
- Two known efficient, NISQ-friendly algorithms:
  - HT – a variant of the Hadamard Test

    [Johri, Steiger, Troyer 17]
  - **TCT** – a variant of the Two-Copy Test

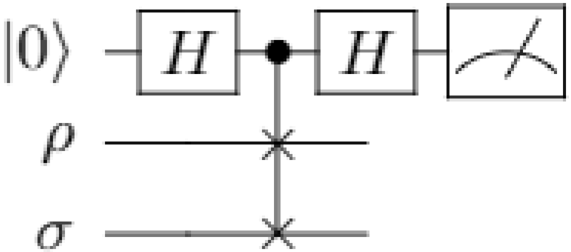
    [Subaşı, Cincio, Coles 19]

# Previous algorithm #1 for $\mathrm{Tr}(\rho_A^n)$ The HT

- Recall SWAP Test:



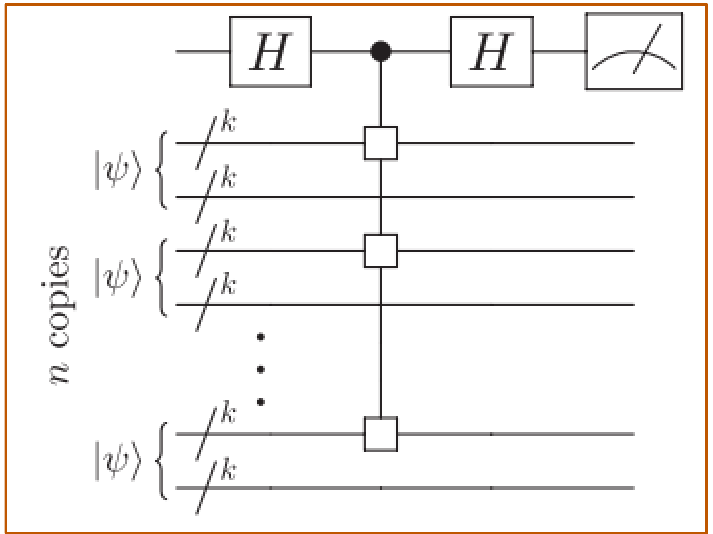

Hadamard Test
$|\langle\psi|M|\psi\rangle|^2$




When $\rho = \sigma$, gives

$$\mathrm{Tr}(\mathrm{SWAP}\rho \otimes \rho) = \mathrm{Tr}(\rho^2)$$

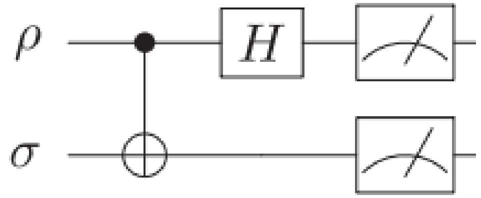
- Generalize [Johri, Steiger, Troyer 2017]:
  Use Cyclic Permutation operator $P_A^{\mathrm{cyc}}$ on $n$ copies of $|\psi\rangle$ to compute $\mathrm{Tr}(\rho_A^n)$.

**HT =** 


Cyclic Permutation:
e.g. $1234 \rightarrow 4123$

# Previous algorithm #2 for $\mathrm{Tr}(\rho_A^n)$
# The TCT

- Bell Basis $(|\phi^+\rangle, |\phi^-\rangle, |\psi^+\rangle, |\psi^-\rangle)$ is an eigenbasis of SWAP.
- A Bell Basis Measurement: a CNOT, an H, and classical postprocessing.




→[Garcia-Escartin, Chamorro-Posada 2013] [Cincio, Subaşı, Sornborger, Coles 2018]
Can measure state overlap using a Bell Basis Measurement.

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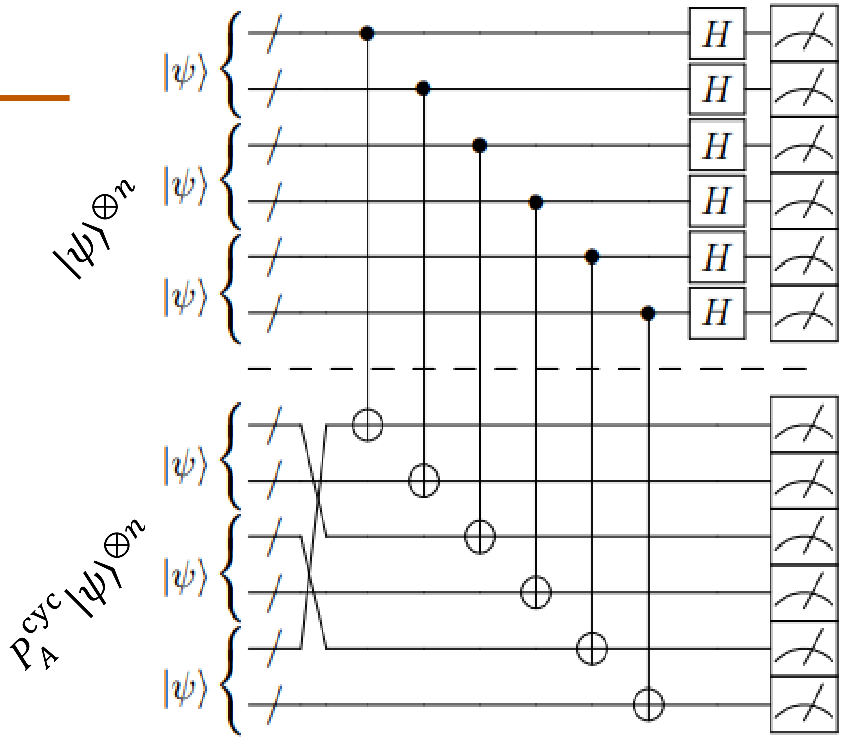
- [Subaşı, Cincio, Coles 2019]
Measure overlap of $|\psi\rangle^{\otimes n}$ and $P_A^{\text{cyc}}|\psi\rangle^{\otimes n}$

$$\left|\langle\psi|^{\otimes n} P_A^{\text{cyc}} |\psi\rangle^{\otimes n}\right|^2 = \mathrm{Tr}(\rho_A^n)^2$$

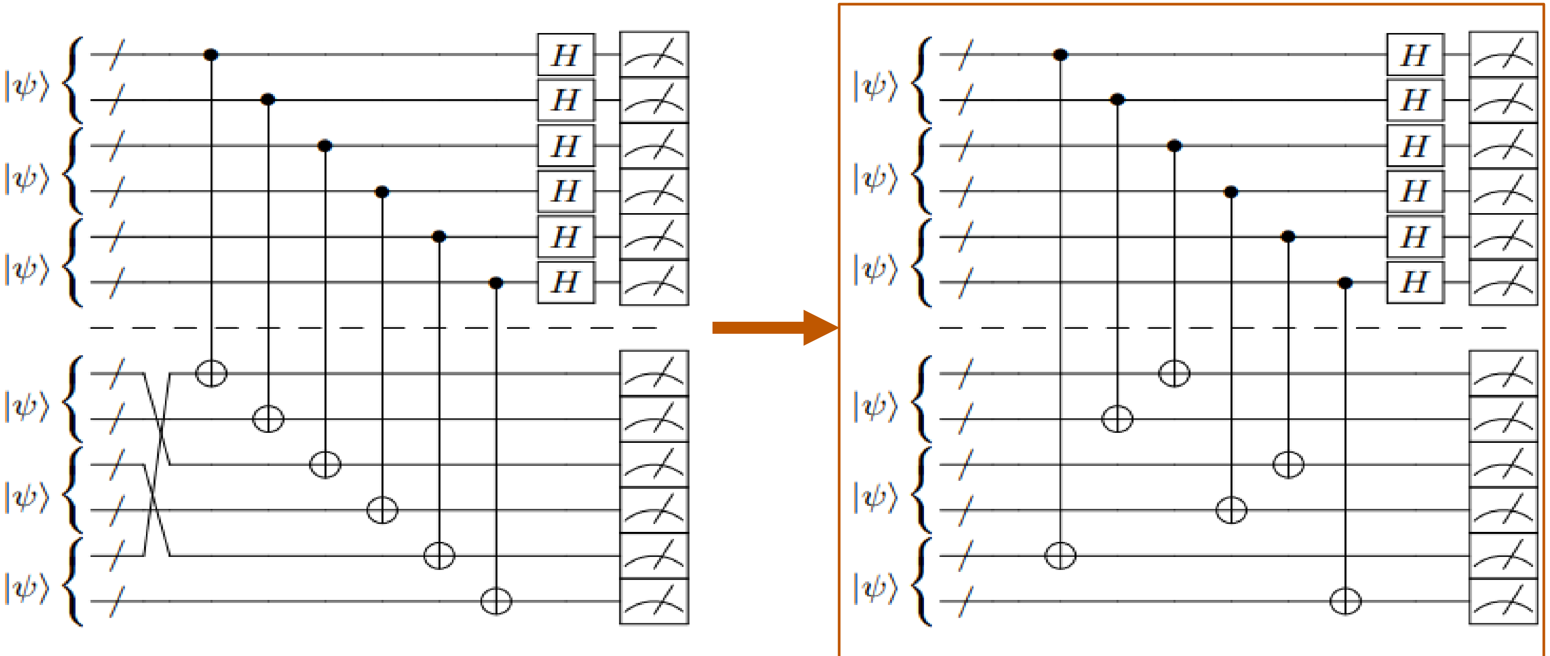
$$\to \mathrm{Tr}(\rho_A^n)$$



# Previous algorithm #2 for $\mathrm{Tr}(\rho_A^n)$ The TCT

[Subaşı, Cincio, Coles 2019] Using Bell-basis Measurement to measure overlap of $|\psi\rangle^{\otimes n}$ and $P_A^{\mathrm{cyc}}|\psi\rangle^{\otimes n}$ to estimate $\mathrm{Tr}(\rho_A^n)$

- Neat trick: Apply $P_A^{\mathrm{cyc}}$ without any gates. Just reindex the CNOTs and the postprocessing formula.
- $O(1)$ depth: 1 layer of CNOT, 1 layer of H



# Our qubit-efficient algorithms

- Where $|\psi\rangle$ is $2k$ qubits,
  - HT requires $2nk = O(nk)$ qubits
  - TCT requires $4nk = O(nk)$ qubits

- We give variants of HT and TCT to compute $\mathrm{Tr}(\rho_A^n)$ that require $O(k)$ qubits (as few as $3k + 1$).
  Independent of $n$. An asymptotic difference.

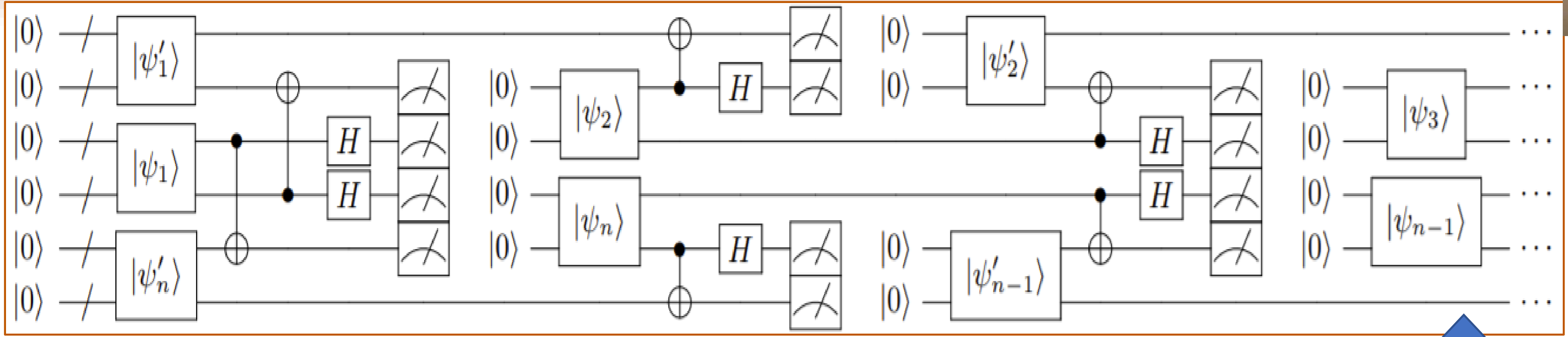
- Our new, low-width circuits have larger depth.
  But, we use qubit resets to avoid the usual noisy affects.

# Qubit resets

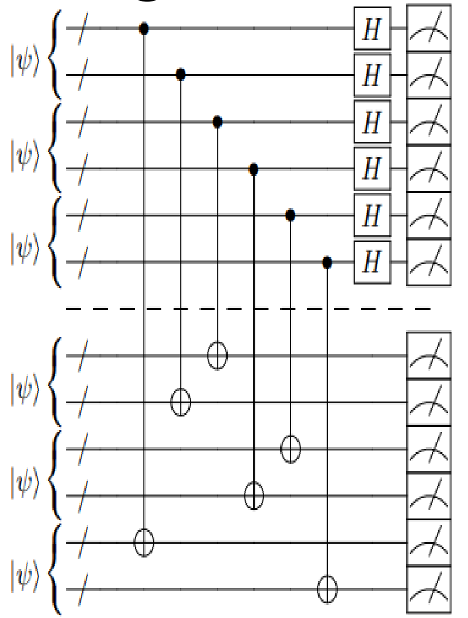
- We usually think of "resetting" qubits to their $|0\rangle$ state at the beginning & end of a computation.
- "Intermediate measurement and reset" is now being rolled out by Honeywell, IonQ, IBM, etc.
- Can reset, i.e. drive back to $|0\rangle$ state, individual qubits in time comparable to a measurement.
  Then, reuse it.
- They're an underexplored tool that will be crucial for NISQ:
  - [Rattew, Sun, Minssen, Pistoia 20] [Foss-Feig et al 20] [Liu, Zhang, Wan, Wang 19] and just a few other examples to date.

# Our algorithm: qe-TCT

**qe-TCT**




**Original TCT**

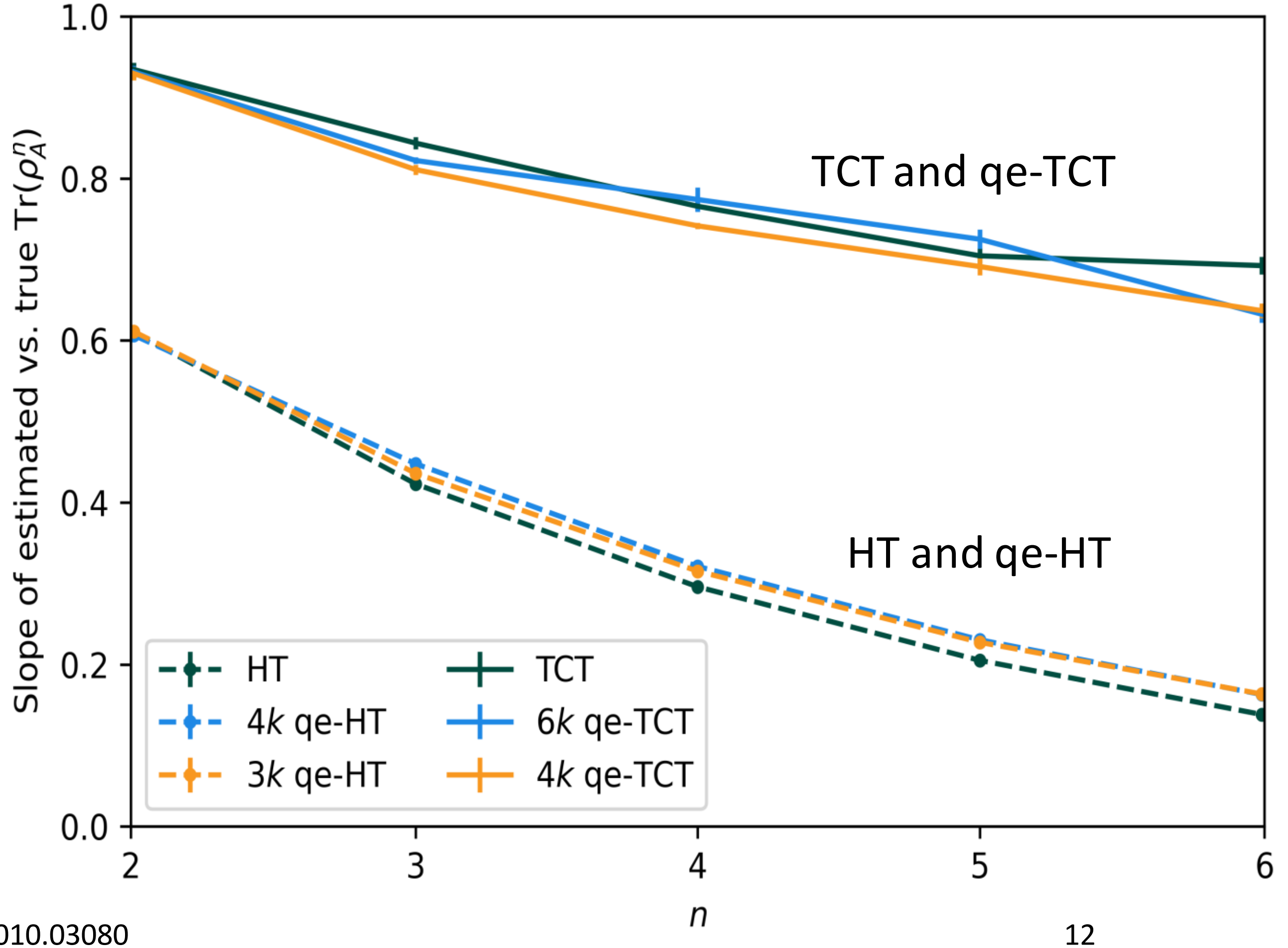



- A new $|0\rangle$ is a qubit reset.
- Each $|\psi_i\rangle$ is identical; indexing just for convenience

- Quick check: each copy only interacts with two other copies.
- **Key intuition**: When a register finishes its interactions, measure, reset, and reuse to load a new copy. Repeat as necessary for $n$.
- So, number of qubits to compute $\mathrm{Tr}(\rho_A^n)$ is independent of $n$

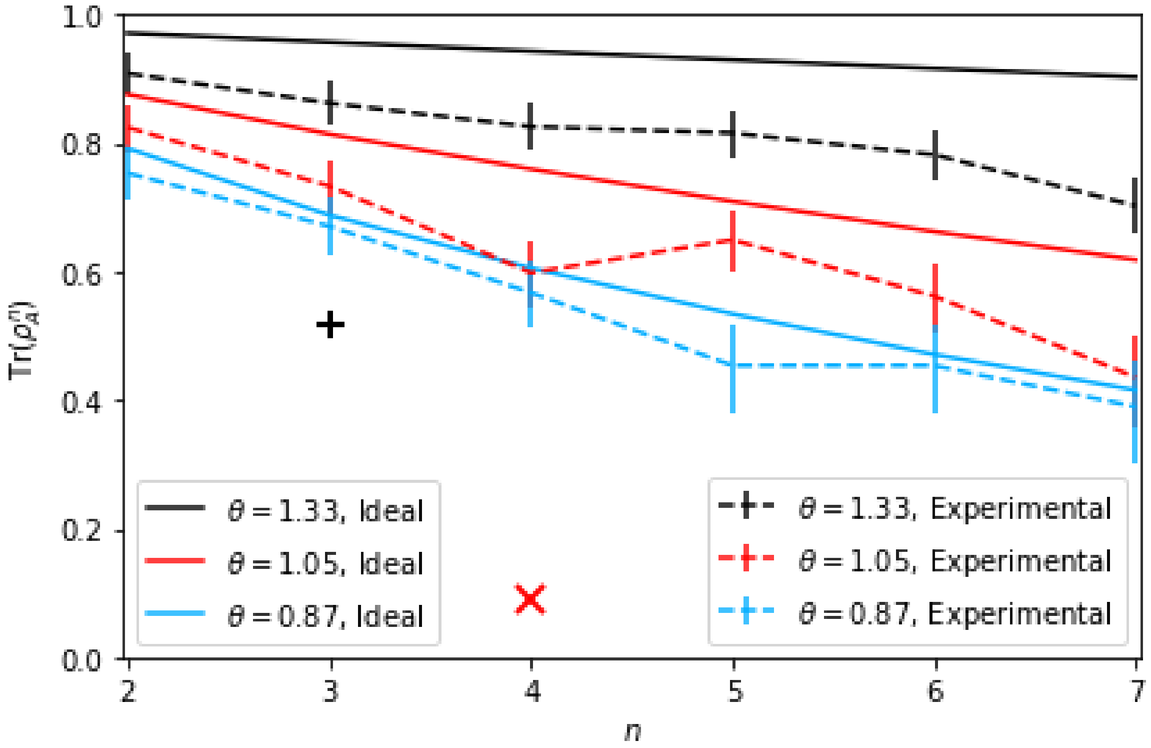
# Numerical simulations

- Tested our algorithms and previous algorithms under simulated gate, thermal, and readout noise (see paper for noise parameters).
- Input different $|\psi\rangle$ with varying entanglement.
  For each value of $n$, use slope of true $\mathrm{Tr}(\rho_A^n)$ vs estimated $\mathrm{Tr}(\rho_A^n)$ to determine quality of algorithm. We plot the slopes.
- **Takeaway**: The new algorithms perform similarly to original.
  HT and qe-HT    TCT and qe-TCT



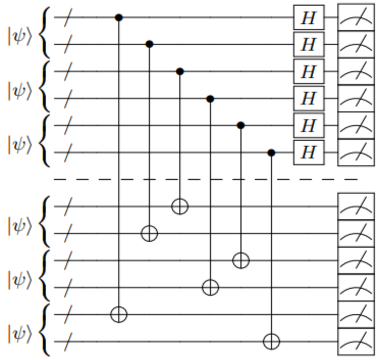
# Honeywell system HØ

- Tested on Honeywell 6-qubit ion trap quantum computer.
- We ran qe-TCT on three different states up to $n = 7$.
  The original TCT would require 28 qubits, more than the 6 available.



# Effective depth

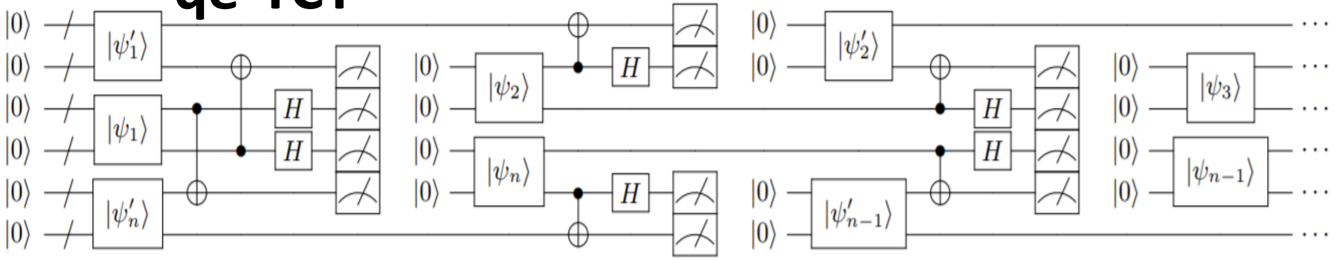
- Circuit depth is interesting because:
  1. Able to prove things about bounded-depth circuits like $\mathrm{AC}_0$, $\mathrm{QNC}_0$, etc.
  2. It's a good heuristic for susceptibility to thermal relaxation and decoherence noise: i.e., deeper circuits perform worse.
- Depth is a bad heuristic for circuits using resets.
  - Evidence: Our circuits!



**Original TCT**

- Original TCT has $O(1)$-depth
- qe-TCT has $\widetilde{\Theta}(n)$-depth
- Asymptotic difference, but they perform similarly.

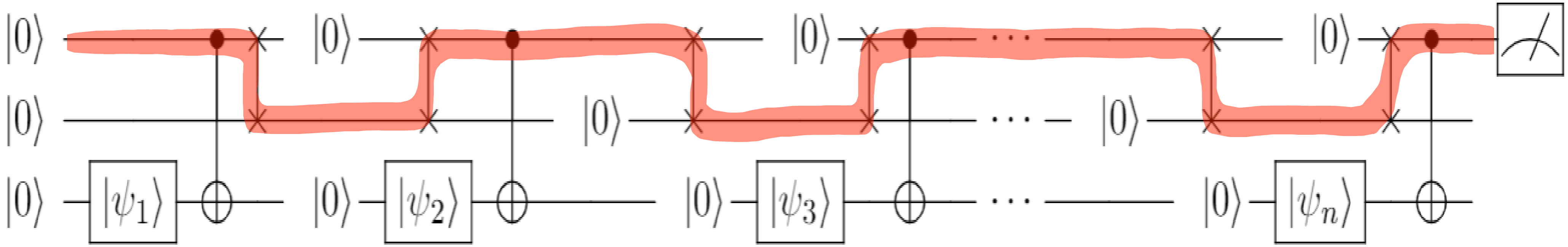
**qe-TCT**



# Effective depth

- Naïve idea: longest time any qubit goes between resets

- Counterexample:

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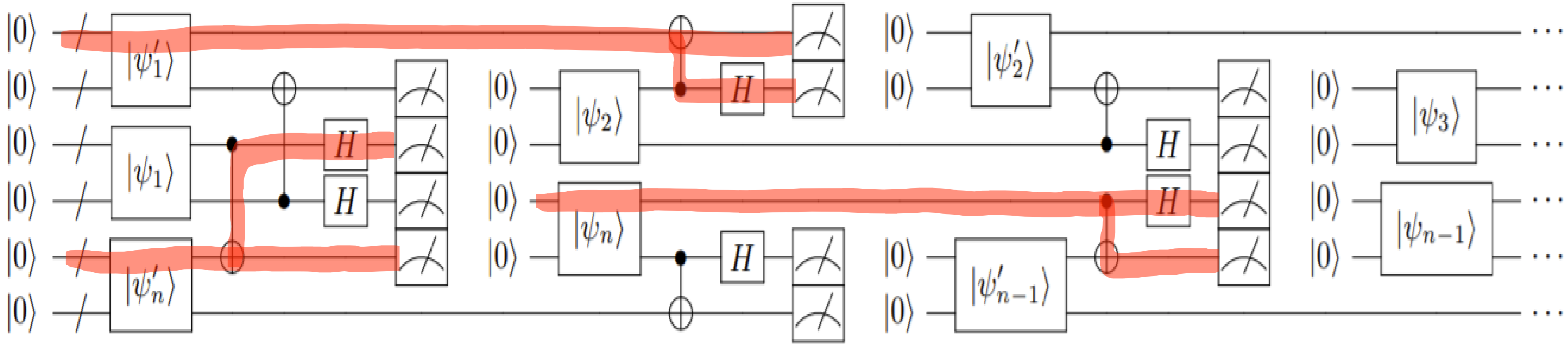
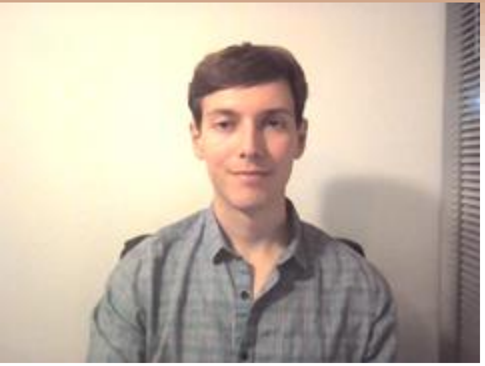
# Effective depth

- Effective circuit depth: maximum length of a path along which there is information flow.
- Then, both the Original TCT and qe-TCT have *effective depth* equal to $O(1)$.




- Reduces to standard depth for circuits without resets.

# Qubit-efficient entanglement spectroscopy using qubit resets



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- New algorithms for estimating $\mathrm{Tr}(\rho_A^n)$ which require asymptotically fewer qubits but achieve similar noise resilience.
  Enable spectroscopy of larger quantum systems on NISQ devices than previously possible.
- *Effective circuit depth* generalizes standard depth to circuits using qubit resets. Useful for predicting noise-resilience of future circuits.
- Open: What other algorithms and applications can be made NISQ-ready using qubit resets?